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Centro de Investigación en Matemáticas (CIMAT), Guanajuato,
México (virtual)
https://rsmeysmm.eventos.cimat.mx/node/1409

Programa de la Sesión Especial

Polinomios Ortogonales y Funciones Especiales

Conferenciantes: Abdon Choque Rivero (UMSNH), Manuel Domínguez de la Iglesia (IMATE UNAM), Lidia Fernández Rodríguez (UGR), Chelo Ferreira González (UNIZAR), Rafael González Campos (UMSNH), Judit Mínguez Ceniceros (UNIRIOJA), Luis Verde Star (UAM).

Organizadores: Luis Enrique Garza Gaona(UCOL), Francisco Marcellán (UC3M) y Miguel Ángel Piñar González (UGR).

Programa (Lunes, 14 de junio de 2021)

- 12:00-13:00 (GTM-5) / 19:00-20:00 (GTM +2):

  - Luis Verde-Star: *Discrete orthogonality of hypergeometric and basic hypergeometric polynomial sequences on linear and quadratic lattices.*

- 13:00-14:00 (GTM-5) / 20:00-21:00 (GTM +2):

  Preguntas y discusión sobre las conferencias grabadas:

  - Judit Mínguez Ceniceros: *Bernoulli-Dunkl and Euler-Dunkl polynomials.*
  - Manuel Domínguez de la Iglesia: *Absorbing-reflecting factorizations for birth-death chains on the integers and their Darboux transformation.*
  - Lidia Fernández Rodríguez: *Procesos de cuasi-nacimiento y muerte.*
  - Chelo Ferreira González: *Aproximaciones asintóticas de las integrales catastróficas.*
  - Abdon Choque-Rivero: *The matrix Toda equations Volterra equations and associated matrix orthogonal polynomials.*
  - Rafael González Campos: *Eigenfunctions and eigenexpansions of integral transforms related to classical orthogonal polynomials.*

Títulos y resúmenes

- Judit Mínguez Ceniceros (Departamento de Matemáticas y Computación. Universidad de La Rioja)

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  Título: *Bernoulli-Dunkl and Euler-Dunkl polynomials.*

  Resumen: Bernoulli-Dunkl and Euler-Dunkl polynomials were introduced in [2] and [3], respectively. In this talk we will show some properties proved in [1] that extend some of the well-known identities in the classical case, such as the Euler-Maclaurin or the Boole summation formulas. We will also study the asymptotic behavior of these polynomials.

  References

  [1] Ó. Ciaurri, J. Mínguez Ceniceros, J.L. Varona, Bernoulli-Dunkl and Euler-Dunkl polynomials and their generalizations, *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM* **113** (2019), no. 3, 2853–2876.

  [2] Ó. Ciaurri, A. Durán, M. Pérez, J.L. Varona, Bernoulli-Dunkl and Apostol-Euler-Dunkl polynomials with applications to series involving zeros of Bessel functions, *J. Approx. Theory* **235** (2018), 20–45.

  [3] A. Durán, M. Pérez, J.L. Varona, Fourier-Dunkl system of the second kind and Euler-Dunkl polynomials, *J. Approx. Theory* **245** (2019), 23–39.

- Manuel Domínguez de la Iglesia (Instituto de Matemáticas. Universidad Nacional Autónoma de México)

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  Título: *Absorbing-reflecting factorizations for birth-death chains on the integers and their Darboux transformation.*

  Resumen: We consider a new way of factorizing the transition probability matrix of a discrete-time birth-death chain on the integers by means of an absorbing and a reflecting birth-death chain to the state 0 and viceversa. First we will consider reflecting-absorbing factorizations of birth-death chains on the integers. We give conditions on the two free parameters such that each of the factors is a stochastic matrix. By inverting the order of the factors (also known as a Darboux transformation) we get new families of "almost" birth-death chains

on the integers with the only difference that we have new probabilities going from the state 1 to the state -1 and viceversa. On the other hand an absorbing-reflecting factorization of birth-death chains on the integers is only possible if both factors are splitted into two separated birth-death chains at the state 0. Therefore it makes more sense to consider absorbing-reflecting factorizations of "almost" birth- death chains with extra transitions between the states 1 and -1 and with some conditions. This factorization is now unique and by inverting the order of the factors we get a birth-death chain on the integers. In both cases we identify the spectral matrices associated with the Darboux transformation, the first one being a Geronimus transformation and the second one a Christoffel transformation of the original spectral matrix. We apply our results to examples of chains with constant transition probabilities. Joint work with C. Juarez.

- Lidia Fernández Rodríguez (Departamento de Matemática Aplicada, Universidad de Granada)

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  Título: *Procesos de cuasi-nacimiento y muerte.*

  Resumen: Los procesos de nacimiento y muerte tienen numerosas aplicaciones y son de gran utilidad cuando se quiere modelar procesos de la naturaleza. El objetivo de este trabajo es relacionar algunos polinomios ortogonales en dos variables con procesos de cuasi-nacimiento y muerte. Nos centraremos especialmente en algunos ejemplos obtenidos mediante la construcción introducida por Agahanov en [1]. En concreto, mostraremos una interpretación probabilística de algunos polinomios en el triángulo, en un dominio parabólico e incluso en un dominio no acotado. Es un trabajo conjunto con Manuel D. de la Iglesia.

  Referencias

  [1] C. A. Agahanov, A method of constructing orthogonal polynomials of two variables for a certain class of weight functions, *Vestnik Leningrad Univ. 20*, no. 19, 5-10. Akhiezer, N. I.,1965.

- Chelo Ferreira González (Departamento de Matemática Aplicada. Universidad de Zaragoza)

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  Título: *Aproximaciones asintóticas de las integrales catastróficas.*

  Resumen: Las integrales catastróficas canónicas simples y dobles

[NIST Handbook of Mathematical Functions, cap. 36], constituyen la herramienta matemática básica de la teoría de las catástrofes, que resume la descripción de fenómenos naturales donde pequeñas perturbaciones del sistema, producen fenómenos abruptos. La asintótica de estas funciones especiales es de momento muy desconocida y poco trabajada, debido al carácter altamente oscilatorio del integrando. En particular, nos centramos en la familia de integrales canónicas simples,

$$\Psi(x_1, \ldots, x_K) := \int_{-\infty}^{\infty} e^{i[u^{K+2} + \sum_{m=1}^{K} x_m u^m]} du$$

para valores grande de una de sus variables $x_1, \ldots, x_K$, y valores fijos de las restantes. A partir de una versión simplificada del método clásico de saddle point, introducida en [1], podemos derivar, en términos de funciones elementales, la aproximación asintótica de estas integrales para valores genéricos de K y de p, siendo x_p la variable asintótica. Igualmente obtenemos las líneas de Stokes. Estos resultados aportan nueva información a la sección 36.11 del mencionado capítulo del "NIST Handbook" dedicada al análisis asintótico de estas integrales.

Referencias

[1] J. L. López, E. Pérez Sinusía and P. Pagola, A systematization of the saddle point method. Application to the Airy and Hankel functions, *J. Math. Anal. Appl.*, **354** (2009), 347-359.

- Abdon Choque-Rivero (Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo)
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  Título: *The matrix Toda equations Volterra equations and associated matrix orthogonal polynomials.*
  Resumen: For $q \times q$ positive measures of the form $e^{-xt}\sigma(dx)$ on $[0, \infty)$ with respect to x and $t \geq 0$, we derive the matrix Toda equations for the three-term recurrence relation coefficients of the corresponding orthogonal matrix polynomials. Furthermore, relations for the matrix version of the Volterra lattice and associated orthogonal polynomials are attained.

- Rafael González Campos (Facultad de Ciencias. Universidad Michoacana)
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Título: *Eigenfunctions and eigenexpansions of integral transforms related to classical orthogonal polynomials.*

Resumen: Explicit quadratures for integral transforms generated by bilinear functions of classical orthogonal polynomials have been found in previous work. In order to distinguish these transforms from others, we cal them the z-Fourier (in the Hermite case), z-Bessel (Laguerre), and z-Appel (Jacobi) transforms. Here, the prefix z means dependence on a complex variable. In each case, the asymptotic form of certain kernel matrix generates a real measure that gives rise to these integral transforms. The real measure also generates the Fourier coefficients of the orthogonal expansion of the integral transforms in terms of the corresponding polynomials. We show how the bilinearity of the kernel matrices can be used to find the eigenfunctions and eigenvalues of these integral transforms.

• Luis Verde-Star (Departamento Matemáticas, Universidad Autónoma Metropolitana)
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Título: *Discrete orthogonality of hypergeometric and basic hypergeometric polynomial sequences on linear and quadratic lattices.*

Resumen: Let $\{u_k(t) : k \geq 0\}$ be a sequence of hypergeometric or basic hypergeometric orthogonal polynomials associated with a sequence of nodes x_j in a linear or quadratic lattice. We will describe a procedure to obtain sequences r_j of weights associated with the nodes x_j in the complex plane that allow us the express the orthogonality of $\{u_k(t)\}$ in the form

$$\langle u_k, u_n \rangle = \sum_{j=0}^{\infty} r_j \, u_k(x_j) u_n(x_j) = \delta_{k,n} \rho_n.$$

where the ρ_n are nonzero numbers.

The weights r_j are determined by a system of equations of the form

$$m_k = \sum_{j=0}^{\infty} r_j v_k(x_j), \qquad k \geq 0,$$

where the m_k are generalized moments of the sequence $\{u_k(t)\}$, and $\{v_k(t)\}$ is the sequence of Newton polynomials associated with the nodes x_j. The matrix of coefficients $[v_k(x_j)]$ is a lower triangular

infinite matrix and it is invertible if the x_j are pairwise distinct. The weights r_k are given by

$$r_k = \sum_{j=k}^{\infty} \frac{m_j}{v'_{j+1}(x_k)}, \qquad k \geq 0.$$

We will present some examples of polynomial sequences for which this series converges for each $k \geq 0$.